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Innovatively Ranking Fuzzy Numbers with Left-Right Areas and Centroids

Research Article

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Abstract. Fuzzy set theory, extensively applied in several fields, has been recognized as a powerful tool in dealing with the knowledge of imprecision due to its ability in representing uncertainty and vagueness mathematically. In fuzzy data analysis, searching for a general measure that can effectively and efficiently rank fuzzy numbers for critical information revelation and decision-making has well attracted the special attention of numerous scholars. Several approaches have been proposed up to date; however, their certain shortcomings spare capacity for enhancement.In this paper, an innovative ranking index incorporating three key components such as left-right areas, expectation value of centroid, and level of optimism is proposed. Through numerically comparative studies thorough with current major ranking methods, our approach demonstrates a significant improvement in terms of ranking robustness and discrimination power.

Keywords. Fuzzy ranking method; Left and right areas; Expectation value of centroid; Optimism level, Fuzzy number

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1. Introduction

Fuzzy set theory has been recognized as a powerful tool in dealing with the knowledge of imprecision due to its special ability in representing uncertainty andvagueness mathematically [1, 2]; thus, it has been successfully employed in several fields in practice. In the area of fuzzy decision-making and data analysis, how to effectively and efficiently rank fuzzy numbers has been attracting special attention of many scholars because ranking fuzzy numbers plays a vital role in providing prerequisite procedures for decision makers [2–4]. Therefore, over the last few decades, more than forty ranking methods have been proposed as in [5,6].

However, an issue of inconsistency among different methods, meaning that different ranking methods often result in disparate ranking orders, is still an open topic [6]. It was found that some methods can only be used in some circumstances which require the membership functions have certain properties, for example, normalized, convex, triangular, trapezoidal, etc., [7]. Also, certain methods have unsatisfactory discrimination ability, or lead to counterintuitive orderings, and sometimes even fail to rank special cases of fuzzy numbers correctly [5,8]. To overcome these shortcomings, this paper therefore proposes a novel index whose ranking discriminatory power is magnified by incorporating centroids of fuzzy numbers into their left and right integral areas. We also take the optimism level into consideration to provide certain flexibility in the decision-making procedure. By investigating some numerical examples used in previously published researches, we compare the ranking results obtained from our proposed method and some existing ones to illustrate our superiority in the ranking robustness and discrimination power.

2. Our Proposed Ranking Method

In this section, we first introduce two fundamental definitions about Left-Right and expectation value of centroid before proposing our ranking index. First, let's consider n fuzzy numbers $\widetilde{A}_i = (a_i, b_i, c_i, d_i; w'_i)$ $(i = \overline{1,n})$; $a_{\min} = \min\{a_1, a_2, \ldots, a_n\}$; $d_{\max} = \max\{d_1, d_2, \ldots, d_n\}$.

Definition 1 (Left-Right areas)**.** The left area of fuzzy number $\widetilde{A}_i$, denoted by $S^L_{\widetilde{A}_i}$, from the $a_{\min}$ to $g^L_{\widetilde{A}_i}(y)$, the inverse function of the left membership function $\xi^L_{\widetilde{A}_i}(x)$ is defined by: $S^L_{\widetilde{A}_i} = \int_0^{w_i} \left[g^L_{\widetilde{A}_i}(y) - a_{\min}\right] dy$ and, similarly, its right area, denoted by $S^R_{\widetilde{A}_i}$, from $g^R_{\widetilde{A}_i}(y)$, the inverse function of the right membership function $\xi^R_{\widetilde{A}_i}(x)$, to $d_{\max}$ is defined by: $S^R_{\widetilde{A}_i} = \int_0^{w_i} \left[d_{\max} - g^R_{\widetilde{A}_i}(y)\right] dy$. $S^L_{\widetilde{A}_i}$ and $S^R_{\widetilde{A}_i}$ are visually plotted in Figure 1 [1,9,10].

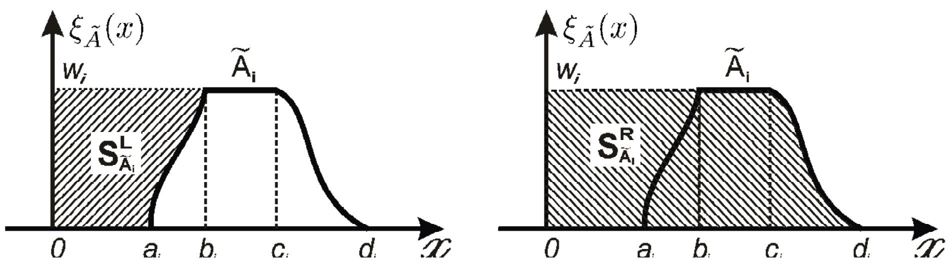

Figure 1. Left and Right areas of fuzzy number $\widetilde{A}_i$

Definition 2 (Expectation value of centroid)**.** An expectation value of centroid of $\widetilde{A}_i$, denoted by EC_i, is defined as [1,2,11]:

$$EC_i = \int_{a_i}^{d_i} x\xi_{\widetilde{A}_i}(x)dx \Big/ \int_{a_i}^{d_i} x\xi_{\widetilde{A}_i}(x)dx.$$

This paper also incorporates decision-maker's attitude towards risk into a solid ranking index to provide critical flexibility and participation of the decision-maker. The attitude used in this paper is denoted by λ, called "*optimism level*"; and $\lambda \in [0,1]$ where 0 indicates "*pessimistic attitude*", 1 indicates "*optimistic attitude*", and 0.5 indicates "*neutral attitude*".

Basing on the $S^L_{\widetilde{A}_i}$, $S^R_{\widetilde{A}_i}$, EC_i and λ defined above, our proposed ranking index for ith number, denoted by $LRAC_i$ is defined as $LRAC_i = \left[\lambda S^R_{\widetilde{A}_i} + (1-\lambda) S^L_{\widetilde{A}_i}\right] EC_i$.

Two fuzzy numbers $\widetilde{A}_i$ and $\widetilde{A}_j$ are then ranked based on the following rules.

- $\widetilde{A}_i \succ \widetilde{A}_j$ if and only if $LRAC_i > LRAC_j$.
- $\widetilde{A}_i \prec \widetilde{A}_j$ if and only if $LRAC_i < LRAC_j$.
- $\widetilde{A}_i \simeq \widetilde{A}_j$ if and only if $LRAC_i = LRAC_j$.

Notably, the expectation value of centroid EC_i used in $LRAC$ index magnifies its ranking discriminatory power, which becomes more efficient than the most recent method proposed by Yu and Dat [10] as illustrated in the following examples.

3. Comparative Examples

Example 1. Consider two L-R fuzzy numbers in Figure 2, $\widetilde{A}_1 = (6,6,1,1)_{LR}$, and $\widetilde{A}_3 = (6,6,0,1)_{LR}$, taken from [4]. Their ranking results obtained from our proposed approach are shown in Table 1 from which we can firmly conclude that $\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3$ regardless of optimism levels of decision-makers (for all $\lambda \in [0,1]$).

Table 1. Ranking results at different optimism levels in Example 1.

λ	$LRAC_{\widetilde{A}_1}$	$LRAC_{\widetilde{A}_2}$	$LRAC_{\widetilde{A}_3}$
0.0	3.00	5.98	6.33
0.1	3.00	5.70	6.01
0.2	3.00	5.41	5.70
0.3	3.00	5.13	5.38
0.4	3.00	4.85	5.06
0.5	3.00	4.56	4.75
0.6	3.00	4.28	4.43
0.7	3.00	4.00	4.11
0.8	3.00	3.71	3.80
0.9	3.00	3.43	3.48
1.0	3.00	3.15	3.16

Our finding is similar to that in [4,12–14] and especially, consistent with that of Yu et al. [1] in ranking both of the fuzzy numbers and their corresponding images; whereas, Chu and Tsao [11] ranked $\widetilde{A}_1 \prec \widetilde{A}_3 \prec \widetilde{A}_2$ and Cheng [15]'s method resulted in $\widetilde{A}_1 \succ \widetilde{A}_2 \succ \widetilde{A}_3$, which were declared unreasonable and inconsistent with human intuition [4,12]. Moreover, Yu and Dat [10]'s method fails to provide sufficient discrimination for an optimistic decision maker. Thus, our approach provides a better discrimination power. Besides, in the next example, Example 2, our approach also outperforms that of Yu and Dat [10] in term of efficiency.

Example 2. Analyze two normal triangular fuzzy numbers $\widetilde{A}_1 = (1,4,5)$ and taken from Yu and Dat [10] as shown in Figure 3. Their $LRAC$ values are respectively found as $LRAC_{\widetilde{A}_1} = 5.00$ and $LRAC_{\widetilde{A}_2} = 5.50$ for all $\lambda \in [0,1]$, meaning that $\widetilde{A}_1 \prec \widetilde{A}_2$ regardless of optimism levels. It can be noted that the ranking result is consistent with that of Yu and Dat [10] whose approach, however, needs extra effort in identifying median values (Me) before the two fuzzy numbers can be ranked. As a matter of fact, the procedure to obtain Me is somewhat complicated as demonstrated by [16,17], and among others. Thus, our approach is more efficient.

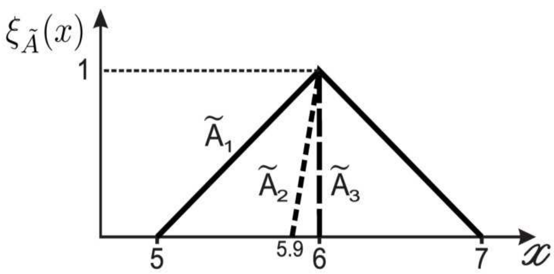

Figure 2. Fuzzy numbers in Example 1

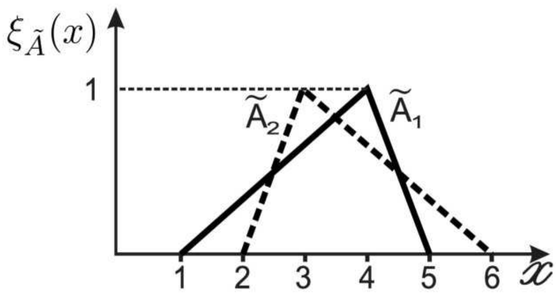

Figure 3. Fuzzy numbers in Example 2

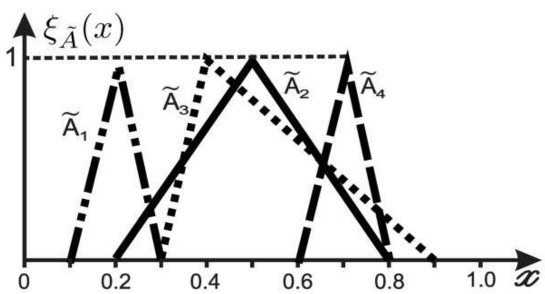

Figure 4. Fuzzy numbers in Example 3

Example 3. Consider four normal fuzzy numbers $\widetilde{A}_1 = (0.1, 0.2, 0.3)$, $\widetilde{A}_2 = (0.2, 0.5, 0.8)$, $\widetilde{A}_3 = (0.3, 0.4, 0.9)$, and $\widetilde{A}_4 = (0.6, 0.7, 0.8)$ in Figure 4. With these numbers, Fortemps and Roubens [18] failed to rank $\widetilde{A}_2$ and $\widetilde{A}_3$, whereas Liou and Wang [19], and Chen and Lu [20] failed to discriminate $\widetilde{A}_1$, $\widetilde{A}_4$ and $\widetilde{A}_2$, $\widetilde{A}_3$. Rao and Shankar [7]'s approach resulted in $\widetilde{A}_1 \prec \widetilde{A}_3 \prec \widetilde{A}_2 \prec \widetilde{A}_4$; nonetheless, their result of $\widetilde{A}_2 \succ \widetilde{A}_3$ is counter-intuitive compared to the discussion in our Example 2. With our proposed index, $LRAC_{\widetilde{A}_2} = 0.125$; $LRAC_{\widetilde{A}_3} = 0.133$; and the four numbers are effectively ranked in Table 2, indicating that our proposed ranking approach can provide better discrimination power and eliminate the counter intuition.

Table 2. Ranking results at different optimism levels in Example 3.

λ	$LRAC_{\widetilde{A}_1}$	$LRAC_{\widetilde{A}_4}$	Ranking result
0.0	0.010	0.385	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.1	0.022	0.357	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.2	0.034	0.329	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.3	0.046	0.301	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.4	0.058	0.273	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.5	0.070	0.245	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.6	0.082	0.217	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.7	0.094	0.189	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.8	0.106	0.161	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \prec \widetilde{A}_4$
0.9	0.118	0.133	$\widetilde{A}_1 \prec \widetilde{A}_2 \prec \widetilde{A}_3 \simeq \widetilde{A}_4$
1.0	0.130	0.105	$\widetilde{A}_4 \prec \widetilde{A}_2 \prec \widetilde{A}_1 \prec \widetilde{A}_3$

Example 4. As showing in Figure 5, we now consider two fuzzy numbers $\widetilde{A}_1 = (1, 2, 5)$ and $\widetilde{A}_2 = (1, 2, 2, 4)$ whose non-linear membership functions are defined as [19]:

$$f_{\widetilde{A}_2} = \begin{cases} \left[1 - (x-2)^2\right]^{\frac{1}{2}}, & x \in [1, 2], \\ \left[1 - \frac{1}{4}(x-2)^2\right]^{\frac{1}{2}}, & x \in [2, 4], \\ 0, & \text{otherwise.} \end{cases}$$

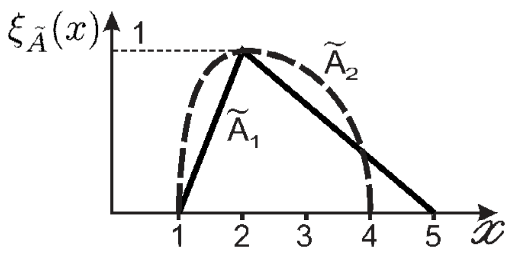

Figure 5. Fuzzy numbers in Example 4

Table 3. Ranking results at different optimism levels in Example 4..

λ	$LRAC_{\widetilde{A}_1}$	$LRAC_{\widetilde{A}_2}$
0.0	1.33	0.52
0.1	1.59	0.81
0.2	1.86	1.10
0.3	2.13	1.40
0.4	2.39	1.69
0.5	2.66	1.99
0.6	2.93	2.28
0.7	3.19	2.58
0.8	3.46	2.87
0.9	3.73	3.17
1.0	3.99	3.46

Table 3 obviously shows that $\widetilde{A}_1 \succ \widetilde{A}_2$ which is similar to those of Nejad and Mashinchi [7], Chu and Tsao [11], Wang et al. [12], Liou and Wang [19], and Ezzati et al. [21], indicating that our proposed approach can effectively rank not only traditional fuzzy numbers but also generalized ones with non-linear membership functions.

4. Conclusion

In fuzzy data analysis and decision-making, a good ranking method provides critical information to make good decisions. Though several approaches have been proposed so far, none of them is widely accepted because each has certain shortcomings to be remedied. In this paper, we proposed an innovative approach by incorporating three critical components: left-right areas, expectation value of centroid, and optimism level, into our ranking index. The integration of the expectation value of centroid grants the special discrimination power of our method. With a comparative study between our proposed approach and some existing prominent ranking methods in four numerical cases, the outstanding performance of our method in ranking generalized fuzzy numbers is affirmed.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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